

Vector, Differential and Integral Identities

1. Vector Identities

$$a \times b = -b \times a$$

$$a \cdot (b \times c) = (a \times b) \cdot c = (c \times a) \cdot b$$

2. Differential Identities:

$$\nabla \times (\nabla p) = \text{curl } \nabla p = 0$$

$$\nabla \cdot (\nabla \times v) = \text{div curl } v = 0$$

$$\nabla \cdot (\phi v) = \nabla \phi \cdot v + \phi \nabla \cdot v$$

$$\nabla \times (\phi v) = \phi \nabla \times v + (\nabla \phi) \times v$$

$$\nabla \times (u \times v) = u (\nabla \cdot v) - (u \cdot \nabla) v + (v \cdot \nabla) u - v (\nabla \cdot u)$$

$$\nabla \times (\nabla \times u) = \nabla (\nabla \cdot u) - \Delta u$$

$$\nabla \cdot (u \times v) = v \cdot \nabla \times u - u \cdot \nabla \times v$$

where $\Delta u = (\Delta u_1, \Delta u_2, \Delta u_3)^T$, $\phi = \phi(x)$ - scal. funct.

3. Vector and Scalar Potentials:

$$\text{div } B = 0 \iff B = \text{curl } A, \quad A - \text{vector potential}$$

$$\text{curl } u = 0 \iff u = \nabla \phi, \quad \phi - \text{scalar potential}$$

! Ass. imposed on Ω : Ω simply connected !

4. Integral Identities:

GAUSS' theorem:

$$\int_V \text{div } u \, dx = \int_{\partial V} u \cdot n \, dS$$

STOKES' theorem:

$$\int_S \text{curl } u \cdot n \, dS = \oint_{\partial S} u \cdot \tau \, ds$$

