4.2.5. The General curl-curl Equation as a Central Research Object

- Let $\Omega \subset \mathbb{R}^3$ a bounded Lip-domain with the boundary $\Gamma = \partial\Omega = \Gamma_D \cup \Gamma_N$.
  Find $u = (u_1, u_2, u_3)^T$ such that

$$
(14) \quad \begin{cases} \operatorname{curl} \mu^{-1} \operatorname{curl} u + \overset{\mathbb{C}}{\varkappa} u = f & \text{in } \Omega, \\ u \times n = g_D & \text{on } \Gamma_D = \Gamma_B, \\ \mu^{-1} \operatorname{curl} u \times n = g_N & \text{on } \Gamma_N = \Gamma_H, \end{cases}
$$

where

$\varkappa = 0$ – for the magnetostatic problem (12), Coulomb gauge: $\operatorname{div} u = 0$

$\mathbb{R}^+ \ni \varkappa = \delta$ (small) – for the conductivity regularization,

$\mathbb{C} \ni \varkappa = i\omega\sigma - \varepsilon\omega^2$ – for the time-harmonic Maxwell eqns.,

$\mathbb{R}^+ \ni \varkappa = \varepsilon/\tau^2 + \sigma/\tau$ – for the implicitely time discr. of (6), (4)

$\mathrm{Im} \ni \varkappa = i\omega\sigma$ – for the time-harmonic eddy current case

$\mathbb{R}^+ \ni \varkappa = \sigma/\tau$ – for the implicitely time discr. of the eddy-curr.

$\mathbb{R}^- \ni \varkappa = -\varepsilon\omega^2$ – for high-frequency time-harmonic appl.

$\mathbb{R}^+ \ni \varkappa = \varepsilon/\tau^2$ – for the impl. time discr. of the wave case