- Under these assumptions, we conclude

$$\underset{\substack{\uparrow \\ \text{BH-relation } (2)_{BH}}}{B =} \begin{pmatrix} B_1(x_1,x_2) \\ B_2(x_1,x_2) \\ 0 \end{pmatrix}, \quad x = (x_1,x_2) \in \Omega$$

$$0 = B_3 \underset{\substack{\uparrow \\ B = \operatorname{curl} A = \nabla \times A}}{=} (\nabla \times A)_3 = \partial_2 A_1 - \partial_1 A_2 = 0$$

This condition is fulfilled with the ansatz

$$A(x) = A(x_1,x_2) = \begin{pmatrix} 0 \\ 0 \\ A_3(x_1,x_2) \end{pmatrix}$$

Furthermore, we automatically have the so-called COLUMB gauche:

$$\operatorname{div} A = \nabla \cdot A = 0,$$

and

$$B = \nabla \times A = \begin{pmatrix} \partial_2 A_3 \\ -\partial_1 A_3 \\ 0 \end{pmatrix}$$

$$|B| = |\nabla \times A| = |\nabla A_3|, \quad \nabla = \begin{pmatrix} \partial_1 \\ \partial_2 \end{pmatrix},$$

$$\operatorname{curl}\Big(\underbrace{\nu(|\operatorname{curl} A|)\operatorname{curl} A}_{= \begin{pmatrix} H_1(x_1,x_2) \\ H_2(x_1,x_2) \\ 0 \end{pmatrix} = \begin{pmatrix} \nu B_1(x_1,x_2) \\ \nu B_2(x_1,x_2) \\ 0 \end{pmatrix}}\Big) = \begin{pmatrix} 0 \\ 0 \\ -\partial_1[\nu \partial_1 A_3] - \partial_2[\nu \partial_2 A_3] \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\operatorname{div}(\nu(|\nabla A_3|)\nabla A_3) \end{pmatrix}$$