4.2.2. Quasistatic Case: The Eddy-Current Problem

- Ass.: $\left|\frac{\partial D}{\partial t}\right| \ll |J|$ ⤵ Displacement currents can be neglected!
  low frequency
- Then we obtain the so-called eddy-current approximation to the full MAXWELL equations:

$$
(8)\left\{
\begin{array}{l}
\operatorname{curl} H = J + \cancel{\frac{\partial D}{\partial t}} \\
\operatorname{div} B = 0 \\
\operatorname{curl} E = -\frac{\partial B}{\partial t} \\
\operatorname{div} D = \rho \\
B = \mu H + \mu_0 M,\ D = \varepsilon E,\ J = \sigma E + \tilde{J}_i \quad (p=0, v=0) \\
+ \text{IFC} + \text{BC} + \text{IC} + \text{RC}
\end{array}
\right.
$$

- E-field based:

TD: $\sigma \frac{\partial E}{\partial t} + \operatorname{curl} \nu \operatorname{curl} E = -\frac{\partial \tilde{J}_i}{\partial t}$ $\quad (\operatorname{div} \varepsilon E = \rho = 0)$

FD: $i\omega\sigma E + \operatorname{curl} \nu \operatorname{curl} E = -i\omega \tilde{J}_i$,

$+ \text{IFC} + \text{BC} + \text{IC} + \text{RC}$

- Vector Potential based: $B = \operatorname{curl} A$

TD: $\sigma \frac{\partial A}{\partial t} + \operatorname{curl} \nu \operatorname{curl} A = \tilde{J}_i := J_i + \operatorname{curl} M$

FD: $i\omega\sigma A + \operatorname{curl} \nu \operatorname{curl} A = \tilde{J}_i$

$+$ IFC : $[A \times n] = 0$, $[\nu \operatorname{curl} A \times n] = 0$ on Γ_I

$+$ BC : $A \times n = 0$ OR $\nu \operatorname{curl} A \times n = 0, \ldots$ on Γ

$+$ IC : $A(x,0) = A_0(x)$, $x \in \overline{\Omega}$,

$+$ RC : SM for $|x| \to \infty$