Vector Potential Formulation:

- $\operatorname{div} B = 0 \Rightarrow \exists$ vector potential A: $B = \operatorname{curl} A$ (3)
- Substituting (3) into Faraday's law $(1)_F$ gives

  (4) $\operatorname{curl}\left(E + \frac{\partial A}{\partial t}\right) = 0.$

  Therefore, there exists a scalar potential φ $-\varphi$

  (5) $E + \frac{\partial A}{\partial t} = -\nabla\varphi$, i.e. $E = -\frac{\partial A}{\partial t} - \nabla\varphi$

  provided that Ω is a bounded, simply connected domain in $\mathbb{R}^3$!
- Hence, Amper's law $(1)_A$ can be expressed by

  $$\operatorname{curl} H \underset{(2)_{BH}}{=} \operatorname{curl}\left(\underbrace{\frac{1}{\mu}}_{=\nu\text{ - reluctivity}} B + \frac{\mu_0}{\mu} M\right) = J + \frac{\partial D}{\partial t}, \text{ i.e.}$$

  $$\operatorname{curl} \nu \operatorname{curl} A \underset{(2)_{DE},\,(2)_{Ohm}}{=} \sigma E + \sigma\, v \times \operatorname{curl} A + J_i + \varepsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} - \operatorname{curl} \frac{\mu_0}{\mu} M$$

  Using (5), we get

  $$(6)\quad \operatorname{curl} \nu \operatorname{curl} A + \sigma\, v \times \operatorname{curl} A + \sigma \frac{\partial A}{\partial t} + \varepsilon \frac{\partial^2 A}{\partial t^2} =$$
  $$= J_i - \operatorname{curl} \frac{\mu_0}{\mu} M + \frac{\partial P}{\partial t} - \sigma \nabla\varphi - \varepsilon \frac{\partial}{\partial t} \nabla\varphi$$

- <u>Exercise 4.1:</u>

  Show that, for any scalar function ψ, the potentials

  $$\tilde{A} = A + \nabla\psi \quad \text{and} \quad \tilde{\varphi} = \varphi - \frac{\partial\psi}{\partial t}$$

  yield the same magnetic and electrical fields, i.e.

  $B = \operatorname{curl} A = \operatorname{curl} \tilde{A}$ and $E = -\frac{\partial A}{\partial t} - \nabla\varphi = -\frac{\partial \tilde{A}}{\partial t} - \nabla\tilde{\varphi}$!