4. Electromagnetics (EM)

4.1. The Full MAXWELL Equations

- Electromagnetical Fields are described by MAXWELL's Equations introduced by James Clerk Maxwell in 1862.

- The electromagnetic quantities involved in Maxwell's equations are: $x \in \Omega \subseteq \mathbb{R}^d$, $t \in [0,T]$

Notation $U = U(x,t)$	Unit SI	Description
$E = (E_1, E_2, E_3)^T$	V/m	electric field intencity
$D = (D_1, D_2, D_3)^T$	As/m^2	electric flux density
$H = (H_1, H_2, H_3)^T$	A/m	magnetic field intensity
$B = (B_1, B_2, B_3)^T$	$Vs/m^2 = T$	magnetic flux density
$J = (J_1, J_2, J_3)^T$	A/m^2	electric current density
$\rho = \rho(x,t)$	As/m^3	electric charge density
$M = (M_1, M_2, M_3)^T$	$Vs/m^2 = T$	magnetitation (permanent)
$P = (P_1, P_2, P_3)^T$	As/m^2	electric polaritation

(E and D related by ε; H and B related by μ; E and J related by σ.)

- The basic differential operators in EM are

$$\operatorname{curl} U = \operatorname{rot} U = \nabla \times U = \det \begin{bmatrix} i & j & k \\ \partial_1 & \partial_2 & \partial_3 \\ U_1 & U_2 & U_3 \end{bmatrix} = \begin{bmatrix} \partial_2 U_3 - \partial_3 U_2 \\ \partial_3 U_1 - \partial_1 U_3 \\ \partial_1 U_2 - \partial_2 U_1 \end{bmatrix}$$

and

$$\operatorname{div} U = \nabla \cdot U = \sum_{i=1}^{3} \frac{\partial U_i}{\partial x_i} = \sum_{i=1}^{3} \partial_i U_i = U_{i,i}$$

where $U = (U_1, U_2, U_3)^T$.