Zugeordnetes Minimumproblem:

(43) Ges. $u \in \bar{V}_0 : J(u) = \inf_{v \in \bar{V}_0} J(v)$,

mit $J(v) = \underbrace{\frac{1}{2} \int_\Omega \sigma_{ij}(v)\, \varepsilon_{ij}(v)\, dx}_{\substack{= \text{Deformationsenergie} \\ \text{(innere Energie)}}} - \underbrace{\left(\int_\Omega f_i v_i\, dx + \int_{\Gamma_t} t_i v_i\, ds\right)}_{\substack{= \text{potentielle Energie} \\ \text{der äußeren Kräfte}}}$

Variationsproblem ⟺ Minimumproblem

(42) ⟺ (43)

Ges. $u \in V_{\bar{u}} := \bar{V}_0$:

$a(u, \delta u) = \langle F, \delta u \rangle$

$\forall$ Variationen $\delta u \in \bar{V}_0$

Ges. $u \in \bar{V}_{\bar{u}} := \bar{V}_0$:

$J(u) = \inf_{\delta u \in \bar{V}_0} J(u + \delta u)$

Euler: $v = \delta u$

$$J(u+\delta u) = \frac{1}{2} a(u+\delta u, u+\delta u) - \langle F, u+\delta u \rangle$$
$$= \frac{1}{2} a(u,u) - \langle F, u \rangle + [a(u,\delta u) - \langle F, \delta u \rangle] + \frac{1}{2} a(\delta u, \delta u)$$
$$= J(u) + [a(u,\delta u) - \langle F, \delta u \rangle] + \frac{1}{2} a(\delta u, \delta u)$$
$$\geq J(u) \quad \forall \delta u \in \bar{V}_0$$

$$\Longleftrightarrow \quad a(u, \delta u) - \langle F, \delta u \rangle = 0 \quad \forall \delta u \in \bar{V}_0$$