Herleitung der Variationsformulierung:

① $V_0 = \{ v=(v_1,v_2,v_3)^T \in V = [W_2^1(\Omega)]^3 : v = 0 \text{ auf } \Gamma_u \}$

② $-\int_\Omega \sigma_{ji,j}\, v_i \, dx = \int_\Omega f_i v_i \, dx \quad \forall v \in V_0$

|| ⇐ partielle Integr.

③ $\int_\Omega \sigma_{ji}\, v_{i,j} \, dx - \int_\Gamma \sigma_{ji} n_j v_i \, ds = \int_\Omega f_i v_i \, dx \quad \forall v \in V_0$

NR: $\sigma_{ji} v_{i,j} = \frac{1}{2}\big(\sigma_{ji} v_{i,j} + \overset{=\sigma_{ji}}{\sigma_{ij}} v_{j,i} \big)$

$= \sigma_{ji} \cdot \frac{1}{2}\big(v_{i,j} + v_{j,i} \big) = \sigma_{ji}(u)\, \varepsilon_{ji}(v)$

$\int_\Omega \sigma_{ji}(u)\, \varepsilon_{ji}(v) \, dx - \int_\Gamma \sigma_{ji} n_j v_i \, ds = \int_\Omega f_i v_i \, dx \quad \forall v \in V_0$

④ $\int_\Gamma \sigma_{ji} n_j v_i \, ds = \int_{\Gamma_u} \sigma_{ji} n_j \underbrace{v_i}_{=0} \, ds + \int_{\Gamma_t} \underbrace{\sigma_{ji} n_j}_{=t_i} \cdot v_i \, ds$

$= \int_{\Gamma_t} t_i v_i \, ds$

$\int_\Omega \sigma_{ji}(u)\, \varepsilon_{ji}(u) \, dx = \int_\Omega f_i v_i \, dx + \int_{\Gamma_u} t_i v_i \, ds \quad \forall v \in V_0$

⑤ $V_g = \{ v \in V : v = \bar{u} := 0 \text{ auf } \Gamma_u \} = V_0$

$\downarrow$

i.A. $\bar{u} \in [H^{1/2}(\Gamma_u)]^3$