

- Für die 8 Ebenen mit den Normalen

$$\vec{n} = (n_1, n_2, n_3)^T : n_i^2 = 1/3, i=1, 2, 3$$

des Oktaeders gilt wegen (12)

$$\sigma_{oct} = \sigma_n = \sigma_i n_i^2 = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) = p$$

$$\begin{aligned} \tau_{oct} &= \tau_n = (\sigma_i n_i^2 - \sigma_n)^{2/3} = \\ &= \left(\frac{1}{3} \sigma_i \sigma_i - \frac{1}{9} (\sigma_1 + \sigma_2 + \sigma_3)^2 \right)^{1/2} \\ &= \frac{1}{3} \left(3(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_1\sigma_3) \right)^{1/2} \\ &= \frac{1}{3} \left(2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3) \right)^{1/2} \\ &= \frac{1}{3} \left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)^{1/2} \\ &= \sqrt{\frac{6}{9} \cdot \frac{1}{6} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} \\ &= \sqrt{\frac{2}{3} (-I_2(\sigma))} = \sqrt{\frac{2}{3} J_2(\sigma)} \\ &= \sqrt{\frac{2}{3}} \tau_I. \end{aligned}$$