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Talk announcement

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A new approach to mixed methods for biharmonic problems in 2D and 3D and efficient solvers for the discretized problems

A new variant of a mixed variational formulation for a biharmonic problem is presented, which involves a non-standard Sobolev space for the Hessian of the original unknown. This allows to rewrite the fourth-order problem as a sequence of three (consecutively to solve) second-order problems. In 2D this decomposition relies on the Hilbert complex

$$H^1(\Omega)^2 \xrightarrow{\text{sym Curl}} \mathbf{H}(\operatorname{div}\operatorname{Div};\Omega,\mathbb{S}) \xrightarrow{\operatorname{div}\operatorname{Div}} L^2(\Omega),$$

in 3D on the Hilbert complex

$$H^1(\Omega)^3 \xrightarrow{\operatorname{dev}\nabla} \mathbf{H}(\text{sym Curl};\Omega,\mathbb{T}) \xrightarrow{\text{sym Curl}} \mathbf{H}(\operatorname{div}\operatorname{Div};\Omega,\mathbb{S}) \xrightarrow{\operatorname{div}\operatorname{Div}} L^2(\Omega),$$

which both are exact for bounded and topologically simple domains, and on a Helmholtz-like decomposition, which is different from the Helmholtz decomposition associated to the Hilbert complexes from above.

On the discrete level this approach can be exploited in 2D either to reformulate the well-known Hellan-Herrmann-Johnson method or to construct a new class of mixed finite element methods for biharmonic problems in such a way that, in both cases, the assembling of the discretized equations involves only standard Lagrangian elements. Similar to the continuous level a decomposition of the discretized problem into three discretized second-order problems is available, which substantially simplifies the construction of efficient solution techniques on the discrete level.

Possible extensions to 3D on the discrete level as well as extensions to more general classes of fourth-order problems will also be shortly discussed.