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Talk announcement

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Nonstandard Discretization Strategies In Isogeometric Analysis of Partial Differential Equations

Isogeometric Analysis (IgA), based on B-spline and Non-Uniform Rational B-Spline (NURBS), is a numerical method proposed in 2005 by Thomas Hughes, John Cottrell and Yuri Bazilevs to approximate solutions of partial differential equations (PDEs). IgA uses the same class of basis functions for both representing the geometry of the computational domain and approximating the solution of problems modeled by PDEs. NURBS basis functions are the main underlying tools in most industrial and engineering design processes. The special properties associated with NURBS including the ease of constructing basis functions with higher smoothness and special refinement strategies makes them a very suitable choice in many real life applications.

In many engineering or practical applications, the computational domains cannot be represented by a single NURBS geometry mapping, and thus must be decomposed into several subdomains. These subdomains are referred to as patches in IgA. In this regards, we developed a multipatch discontinuous Galerkin Isogeometric Analysis (dG-IgA) of PDEs given in

volumetric computational domains as well as on closed and open surfaces. We present the numerical analysis of the dG-IgA schemes proposed, and show a priori error estimates for geometrically matching subdomains with hanging nodes on the interface, i.e., non-matching meshes are allowed.

Many realistic applications involve complex domains with non-smooth boundary parts, changing boundary conditions, non-smooth coefficients arising from material interfaces etc. It is well known that standard numerical schemes on uniform meshes do not yield optimal convergence rate. This is due to the reduced regularity of the solution in the vicinity where the singularities occur.

We therefore develop and analyze a graded mesh technology in isogeometric analysis which leads to the desired and expected optimal convergence rate. The IgA mesh grading uses a priori information of the behavior of the solution around the points, where the singularity occurs, and create an appropriate mesh sequence yielding the same convergence rate as in the smooth case.

Finally, we consider linear parabolic initial-boundary value problems. There are several well-known classical time-stepping schemes for solving parabolic evolution problems like implicit and explicit Runge-Kutta methods. In this thesis, we present a space-time IgA of parabolic evolution problems as an alternative approach to the numerical solution of time-dependent PDE problems. By using time-upwind test functions, we derive a stable space-time IgA scheme that combines very well with parallel solvers. We consider both fixed and moving spatial computational domains. A priori error estimates with respect to some discrete energy norm are presented. Our numerical experiments confirm these theoretical results.