

**Johann Radon Institute for  
Computational and Applied Mathematics  
der  
Österreichischen Akademie der Wissenschaften**

# **Group Seminar**

Group: Computational Methods for Direct Field Problems

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## **“Domain Decomposition for Multiscale PDEs”**

We consider additive Schwarz domain decomposition preconditioners for piecewise linear finite element approximations of elliptic PDEs with highly variable coefficients. In contrast to standard analyses, we do not assume that the coefficients can be resolved by a coarse mesh. This situation arises often in practice, for example in the computation of flows in heterogeneous porous media, in both the deterministic and (Monte-Carlo simulated) stochastic cases. We consider preconditioners which combine local solves on general overlapping subdomains together with a global solve on a general coarse space of functions on a coarse grid. We perform a new analysis of the preconditioned matrix, which shows rather explicitly how its condition number depends on the variable coefficient in the PDE as well as on the coarse mesh and overlap parameters. The classical estimates for this preconditioner with linear coarsening guarantee good conditioning only when the coefficient varies mildly inside the coarse grid elements. By contrast, our new results show that, with a good choice of subdomains and coarse space basis functions, the preconditioner can still be robust even for large coefficient variation inside domains, when the classical method fails to be robust. In particular our estimates prove very precisely the previously made empirical observation that the use of low energy coarse spaces can lead to robust preconditioners. We go on to consider coarse spaces constructed (a) via smoothed aggregation and (b) from multiscale finite elements, and we prove that preconditioners using these types of coarsenings lead to robust preconditioners for a variety of binary (i.e. two-scale) media model problems. Moreover numerical experiments show that the new preconditioners have greatly improved performance over standard preconditioners/solvers even in the random coefficient case. This is joint work with I.G. Graham (University of Bath) and E. Vainikko (University of Tartu, Estonia).

**Thursday, December 21, 2006, 15:30  
Johannes Kepler Universität, HF136**